Factorization of almost periodic matrix functions: some recent results and open problems

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The set AP of (Bohr) almost periodic functions is the closed subalgebra of $L_\infty(\mathbb{R})$ generated by all the exponents $e_\lambda(x) := e^{i\lambda x}$, $\lambda \in \mathbb{R}$. An AP factorization of an n-by-n matrix function G is its representation as a product

$$G = G_+ \text{diag}[e_{\lambda_1}, \dots, e_{\lambda_n}] G_-,$$

where $G_+^{\pm 1}$ and $G_-^{\pm 1}$ have all entries in AP with non-negative (resp., non-positive) Bohr-Fourier coefficients. This is a natural generalization of the classical Wiener-Hopf factorization of continuous matrix-functions on the unit circle, arising in particular when considering convolution type equation on finite intervals, Toeplitz operators with matrix symbols on Hardy or Bezikovitch spaces, etc. The talk will be devoted to the current state of AP factorization theory. Time permitting, problems still open will also be described.